ALLEN PARK HIGH SCHOOL

Summer Assessment

Algebra 2 Summer Packet

For Students Entering Algebra 2

Summer 2015

This summer packet is intended to be completed by the FIRST DAY of school. This packet will be graded and count as the first grade of the marking period. You should be working on the same packet as the class you are taking NEXT YEAR. Feel free to email me if you have any questions regarding your packet by emailing me at: tim.brown@apps.k12.mi.us. We encourage you to check your answers and re-work any problems that were incorrect. **We expect you to spend at least 1 hour each week on your summer math packet**. This packet is not designed for one intense 10 hour session the day before school starts so begin now!

This summer math packet will be worth 50 points and will be your first recorded grade of the marking period**.** Bring your questions and concerns regarding any problems you may have had difficulty with to your class on the first day of school. Start off the year with a great start by completing the packet to the best of your ability.

All Summer Packets can be found on the Math Department Website: http://aphsmath.weebly.com

Allen Park High School Summer Assignment
Algebra 2

Show all work for all problems, regardless of their level of difficulty. Answers will be in the form of positive, negative, whole numbers, fractions and decimals. Leave answers in fraction form unless the question contains decimals.

Order of Operations

Remember PEMDAS (**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally)
You *must* perform the order of operations in a specific order.
P: Parentheses (Grouping symbols: parentheses( ), brackets [ ], braces { }, and fraction bars –).
E: Exponents, such as 3^2.
MD: Multiply OR Divide (compute whatever appears 1st moving from left to right)
AS: Add OR Subtract (compute whatever appears 1st moving from left to right)

Find the value of each expression.

1. $10+16\div4+8$
2. $4\left(3+3^2\right)$
3. $4+2^2-15+4$
4. $\dfrac{14(8-15)}{2}$
5. $7-\left[4+(6\cdot5)\right]$
6. $\left[21-(9-2)\right]\div2$
7. $2.5+3^3-8\div2\cdot4.1$
8. $3+\left[8\div\left(9+2(-4)\right)\right]$

Evaluate each expression if *a* = 5, *b* = 0.25, *c* = ½, *d* = 4. When *b* and *c* are used in the same question, give your answer as a decimal.

9. $d(3+c)$
10. $a+b+d$
11. $a+2b-c$
12. $\dfrac{3ab}{2d}$
13. $\dfrac{3a+4c}{2c}$
14. 2^d+a
15. $2c-4a-5d$
16. $(a+c)^2-bd$

Measures of Central Tendencies

The ***mean*** is the average: Add all the numbers together and divide by how many numbers there are.
The ***mode*** is the most common number: There can be one mode, more than one mode, or no mode at all.
The ***median*** is the middle number when the numbers are arranged from least to greatest.
The ***range*** is the highest number minus the lowest number.

Find the mean, median, mode, and range of each set of data. Round your answers to the nearest tenth.

17. 0, 2, 2, 3, 4
18. 4, 5, 12, 10, 12, 16, 5, 8
19. 4.8, 5.7, 2.1, 2.1, 4.8, 2.1
20. 43, 55, 54, 51, 42, 43, 43

For questions number 21 & 22. The following table lists the number of people killed in traffic accidents over a 10 year period.

Number of Fatalities in Traffic Accidents										
Year	1	2	3	4	5	6	7	8	9	10
Fatalities	959	1037	960	797	663	652	560	619	623	583

21. During this time period, what was the average number of people killed per year?
22. How many people died each day on average in traffic accidents during this time period?

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Classifying Real Numbers

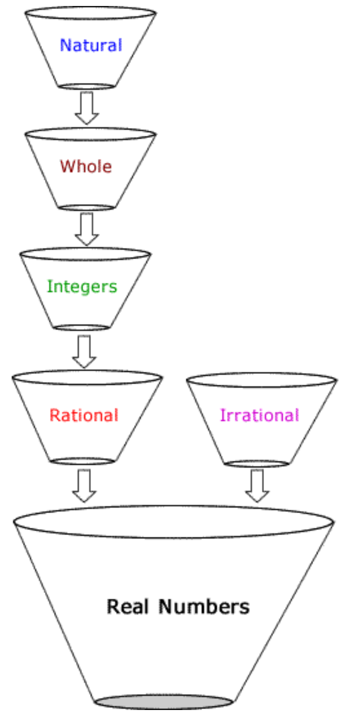
Real numbers can be classified according to a set of characteristics. The diagram below illustrates the classification system for real numbers.

1, 2, 3, 4, …
No fractions, decimals, or negative numbers.

0, 1, 2, 3, 4, …
Includes the natural numbers…no fractions, decimals, or negative numbers.

…-3, -2, -1, 0, 1, 2, 3,…
Includes whole and natural numbers. No fractions or decimals.

$\frac{1}{2}$, -4.15, $8.\overline{3}$, $2\frac{3}{4}$, -8, 0, 4
A number that can be converted to a fraction. This includes integers, whole and natural numbers, as well as fractions and decimals (the decimals must terminate or repeat.)



$\sqrt{37}$, $-6.21459\ldots$, π, $-\sqrt{21}$
Includes numbers that *cannot* be made into fractions, but both contain positive and negative decimals that do *not* repeat or terminate.

All of the numbers you have ever used in mathematics (until later this year).

Classify each number as real $(\mathbb{R})$, rational (Q), irrational (I), whole (W), integer (Z), or natural (N). Some (most) numbers should be classified in more than one category.

23. $2.9+3.7$

24. $-56\div 8$

25. 3^2+2^3

26. $\sqrt{64\div 3}$

27. $\sqrt{25}$

28. $-2\frac{3}{4}+\frac{1}{2}$

29. $4\div 2^3$

30. $\sqrt{36}+2$

Properties of Real Numbers

Property	Addition	Multiplication
Commutative	$a+b=b+a$	$a\bullet b=b\bullet a$
Associative	$(a+b)+c=a+(b+c)$	$(a\bullet b)\bullet c=a\bullet(b\bullet c)$
Identity	$a+0=a$ or $a=0+a$	$a\bullet 1=a$ or $a=1\bullet a$
Inverse	$a+(-a)=0$ or $(-a)+a=0$	$a\bullet\frac{1}{a}=1$ or $\frac{1}{a}\bullet a=1$
Distributive	$a(b+c)=ab+ac$ or $(b+c)a=ba+ca$	

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Name the property illustrated by each equation.

31. $8(4) = 4(8)$
32. $4(m-3) = 4 \bullet m - 4 \bullet 3$
33. $4+0=4$
34. $\frac{5}{4} \bullet \frac{4}{5} = 1$
35. $(5+7)+3 = 5+(7+3)$
36. $(27)3 = 3(27)$
37. $-4+4=0$
38. $(5+9)+13 = 13+(5+9)$
39. $5(-6) = (-6)5$

Simplifying Expressions

Simplify each expression by collecting like terms.

40. $3x+5y+7x-3y$
41. $3a-4c-3c+5a$
42. $2(4c+5d)+6(2c-d)$
43. $3(x-4y)-2(3x-5y)$
44. $7(0.2m+0.3n)+2.4(0.6m-1.2n)$
45. $\frac{1}{2}(4a-2b)+\frac{2}{3}(6b+9a)$

Solving Equations

Remember the general rule: "What you do to one side, you must do to the other." If you add a number to one side of the equation, you must add the same number to the other." This rule holds true for adding, subtracting, multiplying, and dividing numbers or variables to each side of an equation.

Solve.

46. $x+3=4$
47. $-5+b=15$
48. $9-x=78$
49. $-12=4x$
50. $\frac{d}{7}=-8$
51. $9=\frac{3}{7}y$
52. $3c-4=15$
53. $-2f-8=3f-28$
54. $\frac{e+8}{3}=2e-4$
55. $\frac{g}{3}-16=g-4$
56. $3(2a+17)=46$
57. $5=-5(y+3)$
58. $3(4-5k)=2k-4$
59. $2.3n+1=1.3n+7$
60. $\frac{3}{4}n-2=\frac{1}{2}n+7$
61. $3(2c+25)-2(c-1)=78$
62. $-2(x+4)=3(2x-7)+9$
63. $\frac{3}{4}-\frac{1}{5}x=\frac{2}{5}x+\frac{1}{4}$

Working with Formulas

Solve each equation or formula for the specified variable.

64. $I = prt$, for r
65. $A = 2\pi r$, for r
66. $V = lwh$, for h
67. $V = \frac{1}{3}Bh$, for h
68. $4a-5b=8$, for b
69. $qr+s=t$, for q
70. $3x+2y=-4$, for y
71. $y-3=-4(x+1)$, for y
72. $P=2L+2W$, for L

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Absolute Values

Absolute values are a way to ask, "How far away from zero is a given number?" In other words, an absolute value represents a number's distance from zero. Since distance is always positive, when evaluating an absolute value, the answer will positive (unless there is a negative sign *outside* the absolute value).

Evaluate each expression.

73. $|47-62|$
74. $|12-3^2|$
75. $-|8-11|$
76. $|3x+5|$ if $x=-2$
77. $|-4+a|$ if $a=7$
78. $|-8c-4|$ if $c=3$
79. $|2g-1|+1.3$ if $g=4$
80. $-|-x^2|$ if $x=3$

Writing Expressions from Verbal Expressions

Write an algebraic *expression* to represent each verbal expression.

81. a number increased by 15
82. seven decreased by a number
83. fourteen decreased by the square of a number
84. the quotient of a number and two more than 21

Solving Absolute Value Equations

Absolute value equations usually have two different cases when you are trying to solve them. This is illustrated in the following example. If $|x|=4$, what numbers can you substitute into x that will give you a result of 4? There are two different answers; $|4|=4$ and $|-4|=4$, therefore $x=4$ and $x=-4$. Here's another example with an equation:

Solve: $3|2x+3|=15$

Isolate the absolute value first, by dividing both sides by 3 to get $|2x+3|=5$

<u>Case 1</u>: Drop the absolute value on the left
Leave the right side as is

$2x+3=5$ subtract 3 from each side
$2x=2$ divide by 2 on each side
$x=1$

<u>Case 2</u>: Drop the absolute value on the left
Change the right side to negative

$2x+3=-5$ subtract 3 from each side
$2x=-8$ divide by 2 on each side
$x=-4$

Notice, there are two answers (although occasionally, there is only one answer).

Solve each equation.

85. $|x-25|=17$
86. $|k+6|=9$
87. $|3x-7|=18$
88. $2|3x+1|=14$
89. $|4x-8|=0$
90. $|3t-5|=2t$
91. $|a-7|+4=9$
92. $|4a-8|+14=10$

Solving Inequalities

Solve the inequality like an equation. The only rule that is different for inequalities occurs when you multiply or divide by a negative number on both sides you must change the inequality symbol. If it is < and you multiply or divide by a negative number on both sides the inequality will need to change to >.

When graphing you will shade in the portion of the number line that satisfies your answer.
Also, < and > signs result in placing an open circle at the number ($x<3$, put an open circle at three and shade the number line to the left of three).
Lastly, $\le$ and $\ge$ signs result in placing a closed circle at the number ($x \ge -2$, put a closed circle at negative two and shade the number line to the right of negative two).

Solve each inequality. Graph the solution set on a number line.

93. $x>3$

94. $x-7 \le -4$

95. $\frac{1}{2}x+2<1$

96. $x+1 \ge 3x-3$

97. $3 \le \frac{g}{4}-4$

98. $3(4x+7)<21$

99. $2(m-5)+7m>5m+5$

100. $-2n>14$

101. $4-3x \ge 16$

102. $\frac{2x+1}{3}<5$

103. $-2 \le 7-x$

104. $\frac{1}{2}+x>\frac{3}{4}x+\frac{1}{2}$

Writing Equations from Verbal Expressions

Write an algebraic *equation* to represent each verbal expression.

105. eight more than twice a number is five

106. three times the sum or a number and one is 30

Writing Inequalities from Verbal Expressions

Write an algebraic *inequality* to represent each verbal expression.

107. five is less than three times a number

108. four less than a number is greater than two

109. the product of a number and eight is greater than or equal to zero

110. the difference of a number squared and three is less than or equal to 12

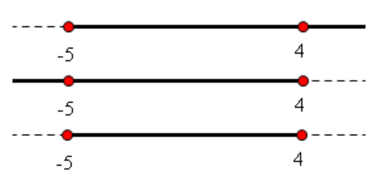
Solving Absolute Value and Compound Inequalities

Combine the rules for solve absolute value *equations* and solving *inequalities*.
You will have *2 cases* like absolute value equations. In case two you must "flip" the inequality symbol *and* change the sign of the side of the inequality without the absolute value symbol.
To graph the solution to an absolute value inequality on a number line, use the following rules:

For < and $\le$, your solution is where the two inequalities will "overlap" one another (the *intersection*).
For > and $\ge$, your solution must include both parts of your solution, (the *union*).
Don't forget about open and closed circles on your graphs.

Example: Solve and graph $|2x+1| \le 9$

Case 1:		Case 2:	
	$2x+1 \le 9$		$2x+1 \ge -9$
	$2x \le 8$		$2x \ge -10$
	$x \le 4$		$x \ge -5$



The last # line is your solution.

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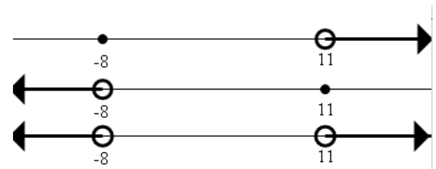
Example: Solve and graph $|2x-3|>19$

Case 1: $2x-3>19$; $2x>22$; $x>11$

Case 2: $2x-3<-19$; $2x<-16$; $x<-8$



The last # line is your solution.

Solve each inequality. Graph the solution set on a number line.

111. $|8a|\le 24$
112. $|2x+4|\ge 7$
113. $|x+2|>5$
114. $x-4\le -7$ or $2x+1>7$
115. $-5<c+2<8$
116. $3|4x-7|<27$
117. $|2x+4|<-9$
118. $x+2<3$ or $-3x-5<7$

Simplifying and Rationalizing Radicals

Example 1: Simplify $\sqrt{8}$

$$\sqrt{8}=\sqrt{4\bullet 2}=\sqrt{4}\bullet\sqrt{2}=2\sqrt{2}$$

Example 2: Simplify $\sqrt{48}$

$$\sqrt{48}=\sqrt{16\bullet 3}=\sqrt{16}\bullet\sqrt{3}=4\sqrt{3}$$

Rationalizing Radicals: You cannot have a *square root* in the denominator of a fraction. You must "rationalize" the denominator in order for the number to be simplified.

Example 3: Simplify $\frac{2}{\sqrt{5}}$

$\frac{2}{\sqrt{5}}$ multiply top and bottom by $\sqrt{5}$

$$\frac{2}{\sqrt{5}}\bullet\frac{\sqrt{5}}{\sqrt{5}}=\frac{2\sqrt{5}}{\sqrt{25}}=\frac{2\sqrt{5}}{5}$$

Example 4: Simplify $\frac{3}{\sqrt{6}}$

$\frac{3}{\sqrt{6}}$ multiply top and bottom by $\sqrt{6}$

$$\frac{3}{\sqrt{6}}\bullet\frac{\sqrt{6}}{\sqrt{6}}=\frac{3\sqrt{6}}{\sqrt{36}}=\frac{3\sqrt{6}}{6}=\frac{{}^{1}\cancel{3}\sqrt{6}}{\cancel{6}_{2}}=\frac{\sqrt{6}}{2}$$

Simplify.

119. $\sqrt{24}$
120. $\sqrt{50}$
121. $\sqrt{27}$
122. $\sqrt{56}$
123. $\sqrt{44}$
124. $\frac{1}{\sqrt{2}}$
125. $\frac{2}{\sqrt{3}}$
126. $\frac{8}{\sqrt{6}}$

Solving Quadratic Equations Using Factoring

Solving Equations by Factoring.

Example 1: Solve. $x^2+3x+2=0$

$\overbrace{x^2+3x+2}^{\text{multiply } 1\text{x}2=2}=0$ — What are the factors of 2 (from 1x2) that add up to the middle term 3?

$x^2+x|+2x+2=0$ — 1 x 2 = 2 $1+2=3$ substitute $1x+2x$ for $3x$

$x(x+1)+2(x+1)=0$ — Pull out the GCF for x^2+x......$x(x+1)$ and for $2x+2$......$2(x+1)$

$(x+2)(x+1)=0$ — Notice the $(x+1)$ is the same for both parts... this becomes ONE factor and the #s outside give you $(x+2)$

$x+2=0$ $x+1=0$ — Set each parenthesis to zero

$x=-2$ $x=-1$ — Solve each equation.

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Example 2: Solve. $3x^2+2x-8=0$

$\overbrace{3x^2+2x-8}^{\text{multiply } 3x-8=-24}=0$ What are the factors of -24 (from $3x-8$) that add up to the middle term 2?

$3x^2+6x\mid-4x-8=0$ $6x-4=-24$ $6+-4=2$ substitute $6x-4x$ for $2x$

$3x(x+2)-4(x+2)=0$ Pull out the GCF for $3x^2+6x......3x(x+2)$ and for $-4x-8......-4(x+2)$

$(3x-4)(x+2)=0$ Notice the $(x+2)$ is the same for both parts...

this becomes ONE factor and the #s outside give you $(3x-4)$

$3x-4=0$ $x+2=0$ Set each parenthesis to zero

$3x=4$ $x=-2$ Solve each equation.

$x=\frac{4}{3}$

Solve using factoring.

127. $x^2+6x+8=0$

128. $x^2-2x-15=0$

129. $2x^2+3x-9=0$

130. $3x^2-2x-8=0$

131. $x^2-7x+12=0$

132. $2x^2+3x-20=0$

Quadratic Formula

Solving Equations Using the Quadratic Formula

The quadratic formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ for a quadratic equation in the form $ax^2+bx+c=0$

Example 1: Solve. $x^2+3x+2=0$

$a=1, b=3, c=2$

$$x=\frac{-3\pm\sqrt{(3)^2-4(1)(2)}}{2(1)} \rightarrow x=\frac{-3\pm\sqrt{9-8}}{2} \rightarrow x=\frac{-3\pm\sqrt{1}}{2}$$

$$x=\frac{-3\pm1}{2} \rightarrow x=\frac{-3+1}{2} \text{ and } x=\frac{-3-1}{2}$$

$$x=\frac{-2}{2} \qquad x=\frac{-4}{2}$$

$$x=-1 \qquad x=-2$$

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Example 2: Solve. $2x^2 - 5x - 7 = 0$

$a = 2, b = -5, c = -7$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(2)(-7)}}{2(2)} \rightarrow x = \frac{5 \pm \sqrt{25+56}}{4} \rightarrow x = \frac{5 \pm \sqrt{81}}{4} \rightarrow x = \frac{5 \pm 9}{4}$$

$x = \frac{5+9}{4}$ and $x = \frac{5-9}{4}$

$x = \frac{14}{4}$ $\quad x = \frac{-4}{4}$

$x = \frac{7}{2}$ $\quad x = -1$

Solve using the quadratic formula.

133. $x^2 + 6x + 9 = 0$

134. $x^2 - x - 12 = 0$

135. $2x^2 + 5x - 12 = 0$

136. $3x^2 - 7x - 6 = 0$

137. $x^2 - 10x + 21 = 0$

138. $2x^2 + 5x - 25 = 0$