# ALLEN PARK HIGH SCHOOL

## Summer Assessment

# Pre-Calculus Summer Packet

## For Students Entering Pre-Calculus

**Summer 2015**

This summer packet is intended to be completed by the FIRST DAY of school. This packet will be graded and count as the first grade of the marking period. You should be working on the packet for the class you are taking NEXT YEAR. Feel free to email me if you have any questions regarding your packet by emailing me at: tim.brown@apps.k12.mi.us. We encourage you to check your answers and re-work any problems that were incorrect. **We expect you to spend at least 1 hour each week on your summer math packet.** This packet is not designed for one intense 10 hour session the day before school starts, so begin now!

**This summer math packet will be worth 50 points and will be the first recorded grade of the marking period.** Bring your questions and concerns regarding any problems you may have had difficulty with to your class on the first day of school, September 4th. Start off the year with a great start by completing the packet to the best of your ability.

**Show all work for all problems, regardless of their level of difficulty. Answers will be in the form of positive, negative, whole numbers, fractions and decimals. Leave answers in fraction form unless the question contains decimals.**

## Solving Absolute Value Equations

Absolute value equations usually have two different cases when you are trying to solve them. This is illustrated in the following example. If $|x| = 4$, what numbers can you substitute into *x* that will give you a result of 4? There are two different answers; $|4| = 4$ and $|-4| = 4$, therefore $x = 4$ and $x = -4$. Here's another example with an equation:

Solve: $3|2x+3| = 15$

Isolate the absolute value first, by dividing both sides by 3 to get $|2x+3| = 5$

Case 1: Drop the absolute value on the left
Leave the right side as is

$2x+3=5$ subtract 3 from each side

$2x=2$ divide by 2 on each side

$x=1$

Case 2: Drop the absolute value on the left
Change the right side to negative

$2x+3=-5$ subtract 3 from each side

$2x=-8$ divide by 2 on each side

$x=-4$

*Notice, there are two answers (although occasionally, there is only one answer).*

**Solve each equation.**

1. $|x-25| = 17$
2. $|k+6| = 9$
3. $|3x-7| = 18$
4. $2|3x+1| = 14$
5. $|4x-8| = 0$
6. $|3t-5| = 2t$
7. $|a-7|+4 = 9$
8. $|4a-8|+14 = 10$

## Solving Absolute Value and Compound Inequalities

Combine the rules for solve absolute value *equations* and solving *inequalities*.
You will have *2 cases* like absolute value equations. In case two you must "flip" the inequality symbol *and* change the sign of the side of the inequality without the absolute value symbol.
To graph the solution to an absolute value inequality on a number line, use the following rules:

For $<$ and $\leq$, your solution is where the two inequalities will "overlap" one another (the *intersection*).
For $>$ and $\geq$, your solution must include both parts of your solution, (the *union*).
Don't forget about open and closed circles on your graphs.

Example: Solve and graph $|2x+1| \leq 9$

Case 1: $2x+1 \leq 9$

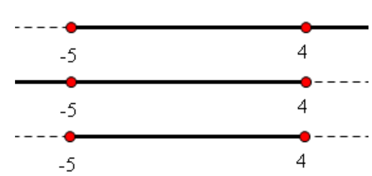
$2x \leq 8$

$x \leq 4$

Case 2: $2x+1 \geq -9$

$2x \geq -10$

$x \geq -5$

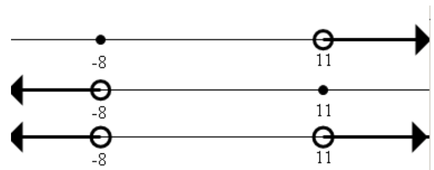


The last # line is your solution.

Example: Solve and graph $|2x-3|>19$

Case 1: $2x-3>19$, $2x>22$, $x>11$

Case 2: $2x-3<-19$, $2x<-16$, $x<-8$



The last # line is your solution.

**Solve each inequality. Graph the solution set on a number line.**

9. $|8a|\le 24$

10. $|2x+4|\ge 7$

11. $|x+2|>5$

12. $x-4\le -7$ or $2x+1>7$

13. $-5<c+2<8$

14. $3|4x-7|<27$

15. $|2x+4|<-9$

16. $x+2<3$ or $-3x-5<7$

## Completing the Square

Example 1: Solve by completing the square $x^2+4x+5=12$

$$x^2+4x+\underset{-5}{5}=\underset{-5}{12}$$

$$x^2+4x+\underset{\left(\frac{4}{2}\right)^2}{\_\_\_\_\_}=7+\_\_\_\_\_$$

$$x^2+4x+\underline{4}=7+\underline{4}$$

$$(x+2)^2=11 \quad \text{factor the left side of the equation}$$

$$\sqrt{(x+2)^2}=\sqrt{11}$$

$$x+2=\pm\sqrt{11}$$

$$x=-2\pm\sqrt{11}$$

**Solve by completing the square.**

17. $v^2-14v-49=-2$

18. $n^2-4n-29=3$

## Equation of a Circle

The equation of a circle: $(x-h)^2+(y-k)^2=r^2$ where $(h,k)$ is the center and $r$ is the radius.

**Use the information provided to write the standard form equation of each circle.**

19. center: $(16,-3)$ radius: 2

20. center: $(-5,4)$ radius: $\sqrt{62}$

**Simplifying Complex Numbers**

Remember: $i^2 = -1$

Example 1: Simplify. $\underbrace{(8i)(-2i)}_{\text{multiply}}(4+9i)$

$\underset{-16\bullet -1}{-16i^2}(4+9i)$

$16(4+9i)$ distribute 16

$64+144i$

Example 2: Simplify.

$\dfrac{-2+3i}{4-5i}$ multiply by the conjugate of the denom.

$\dfrac{-2+3i}{4-5i} \bullet \dfrac{4+5i}{4+5i}$ foil the top and bottom

$\dfrac{-8-10i+12i+15i^2}{16+20i-20i-25i^2}$ simplify

$\dfrac{-8+2i-15}{16+25}$ simplify

$\dfrac{-23-2i}{41}$

**Simplify.**

21. $(-2i)(-3i)(-8-3i)$    22. $(-8+2i)^2$    23. $\dfrac{-5+2i}{3+4i}$    24. $\dfrac{6-i}{-7-3i}$

**Identify the vertex and axis of symmetry of each. Then sketch the graph.**

Axis of Symmetry: $x = \dfrac{-b}{2a}$    Vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

Example 1: Find the vertex and axis of symmetry of $f(x) = 4x^2 + 8x - 7$.

Axis of symmetry: $a = 4, b = 8, c = -7$    $x = \dfrac{-b}{2a} = \dfrac{-8}{2 \bullet 4} = \dfrac{-8}{8} = -1$

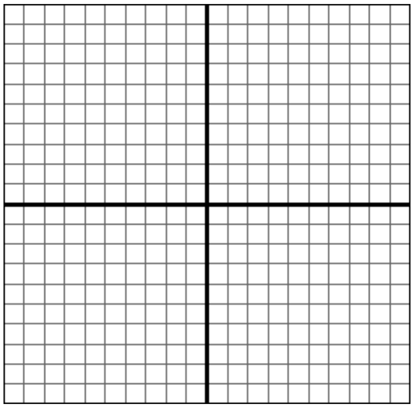
Vertex: Use the *x* value from the axis of symmetry and substitute it into the given equation.

$f(-1) = 4(-1)^2 + 8(-1) - 7 = 4 - 8 - 7 = -11$

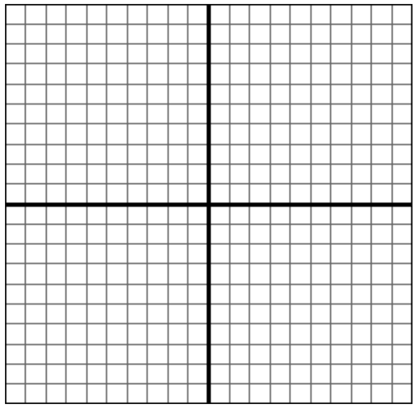
The vertex is at $(-1, -11)$

**Find the vertex and axis of symmetry of each equation. Then sketch the graph.**

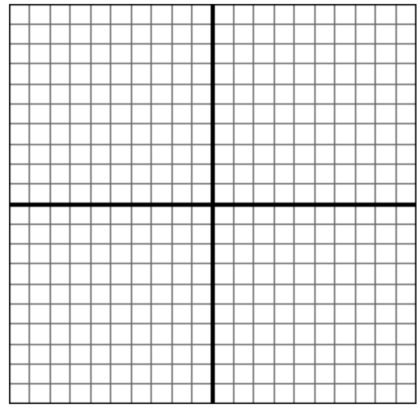
25. $y = -3x^2 + 24x - 52$



26. $f(x) = 3x^2 + 18x + 21$



27. $y = \frac{1}{3}x^2 + 3$



**Divide using synthetic division.**

Example 1: Divide using synthetic division.

$$\left(7x^4 + 2x^2 - 3x - 126\right) \div \left(x + 2\right)$$

$$\begin{array}{r|rrrrr} -2 & 7 & 0 & 2 & -3 & -126 \\ & & -14 & 28 & -60 & 126 \\ \hline & 7 & -14 & 30 & -63 & \boxed{0} \end{array}$$

$7x^3 - 14x^2 + 30x - 63$

With a remainder of 0.

$x + 2$ is a factor of $7x^4 + 2x^2 - 3x - 126$.

Example 2: Divide using synthetic division.

$$\left(x^3 - 2x^2 - 4\right) \div \left(x - 4\right)$$

$$\begin{array}{r|rrrr} 4 & 1 & -2 & 0 & -4 \\ & & 4 & 8 & 32 \\ \hline & 4 & 2 & 8 & \boxed{28} \end{array}$$

$4x^2 + 2x + 8 + \frac{28}{x-4}$

With a remainder of 28.

$x - 4$ is NOT a factor of $x^3 - 2x^2 - 4$.

**Divide.**

28. $\left(x^3 + 9x^2 + 7x - 24\right) \div \left(x + 2\right)$

29. $\left(n^3 - 20n^2 + 108n - 86\right) \div \left(n - 10\right)$

**Solve each equation with the quadratic formula.** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

30. $4p^2 - 34 = -9p$

31. $7x^2 = -6 - 12x$

**Solving Exponential Equations**

Example 1: Solve.

$9^{2x+7} = 27^{x-3}$ make the bases the same...3

$\left(3^2\right)^{2x+7} = \left(3^3\right)^{x-3}$

$3^{2(2x+7)} = 3^{3(x-3)}$ distribute the exponent

$3^{4x+14} = 3^{3x-9}$ since the bases are the same, set the exponents equal to each other.

$4x+14 = 3x+9$ solve for *x*.

$x+14 = 9$

$x = -5$

**Solve each equation.**

32. $25^{3-2v} = 125$

33. $4^{2x+2} = 64$

**Simplifying Monomials**

<u>Example 1:</u> Simplify. Your answer should contain only positive exponents.

$\frac{3a^4b^{-2}}{6a^{-2}b^5}$ reduce the fraction $\frac{3}{6}$

$\frac{{}^{1}\not{3}a^4b^{-2}}{{}_{2}\not{6}a^{-2}b^5}$

$\frac{a^4b^{-2}}{2a^{-2}b^5}$ subtract exponents of common bases

$\frac{a^{4-(-2)}b^{-2-5}}{2}$

$\frac{a^{4+2}b^{-7}}{2}$

$\frac{a^6b^{-7}}{2}$ since *b* has a negative exponent move it to the denominator

$\frac{a^6}{2b^7}$ *b* now has a positive exponent.

Example 2: Simplify. Your answer should contain only positive exponents.

$\dfrac{(2a^2b^{-1})\bullet(a^{-3}b^4)}{4a^4b^{-2}}$ numerator: multiply the coefficients and add exponents of common bases

$\dfrac{2a^{2+(-3)}b^{-1+4}}{4a^4b^{-2}}$

$\dfrac{2a^{-1}b^3}{4a^4b^{-2}}$ reduce the coefficients

$\dfrac{{}^1\not{2}\,a^{-1}b^3}{{}_2\not{4}\,a^4b^{-2}}$

$\dfrac{a^{-1}b^3}{2a^4b^{-2}}$ subtract exponents of common bases

$\dfrac{a^{-1-4}b^{3-(-2)}}{2}$

$\dfrac{a^{-5}b^5}{2}$ move $a^{-5}$ to the denominator

$\dfrac{b^5}{2a^5}$

**Simplify. Your answer should contain only positive exponents.**

34. $\dfrac{ba^{-1}}{a^{-2}b^3\bullet(ab^4)^2}$

35. $\dfrac{x^{-1}y^{-3}\bullet xy^3}{(2x^4)^{-4}}$

36. $\dfrac{(m^4n^2)^3\bullet(m^{-3})^3}{(2n^{-3})^4}$

**Factoring Quadratics**

Example 1: Factor completely. $x^3+3x^2+2x$

$x^3+3x^2+2x$ Factor out the GCF $x$

$x(x^2+3x+2)$

$x(x^2+3x+2)$ (multiply 1x2=2) What are the factors of 2 (from 1x2) that add up to 3

$x(x^2+x|+2x+2)$ 1x2 = 2 1+2 = 3 substitute $1x+2x$ for $3x$

$x(x(x+1)+2(x+1))$ Pull out the GCF for $x^2+x...x(x+1)$ and for $2x+2...2(x+1)$

Notice the $(x+1)$ is the same for both parts...

$x(x+2)(x+1)$ this becomes ONE factor and the #s outside the ( ) give you $(x+2)$

Example 2: Factor completely. $3x^2+2x-8$

multiply $3x-8=-24$

| | |
|---|---|
| $3x^2+2x-8$ | What are the factors of $-24$ (from $3x-8$) that add up to the middle term 2? |
| $3x^2+6x \mid -4x-8$ | $6 \times -4 = -24$ $6 + -4 = 2$ substitute $6x-4x$ for $2x$ |
| $3x(x+2)-4(x+2)$ | Pull out the GCF for $3x^2+6x$......$3x(x+2)$ and for $-4x-8$......$-4(x+2)$ |
| $(3x-4)(x+2)$ | Notice the $(x+2)$ is the same for both parts... |
| | this becomes ONE factor and the #s outside give you $(3x-4)$ |

**Factor Completely.**

37. $21x^4+78x^3-135x^2$

38. $5n^4-36n^3+36n^2$

39. $28x^2+4x$

40. $4n^3-6n^2$

41. $a^2-4$ difference of squares

42. $k^2-2k+1$

43. $25x^2-16$ difference of squares

44. $36n^2-9$ difference of squares

45. $x^3+8$ sum of cubes

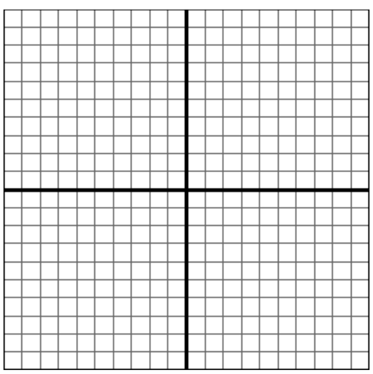
46. $3a^3-81$ gcf/difference of cubes

47. $a^3+27$ sum of cubes

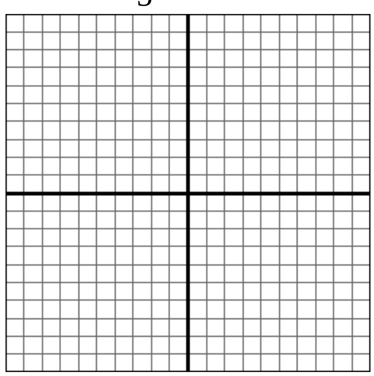
48. $125+64x^3$ sum of cubes

**Sketch the graph of each line or inequality.**

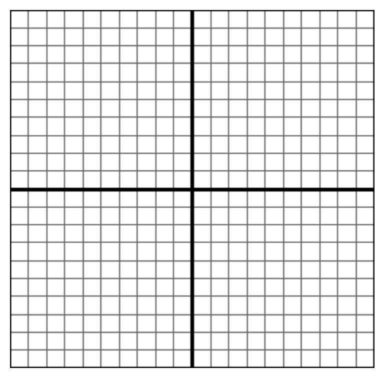
49. $y=-9x+5$



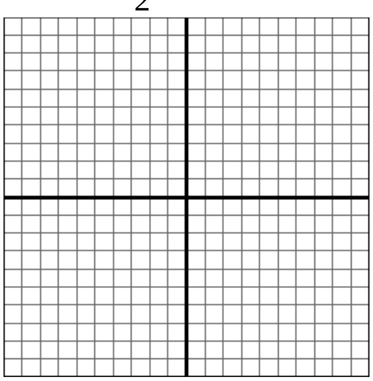
50. $y=-\frac{7}{5}x+2$



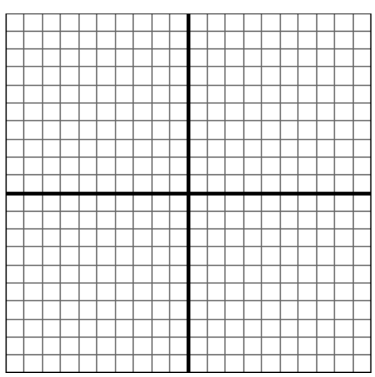
51. $3x-4y=12$



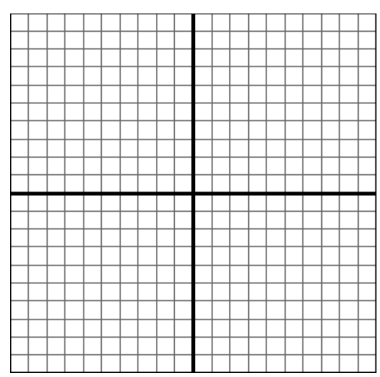
52. $y \geq -\frac{1}{2}x-5$



53. $2x+3y<-9$

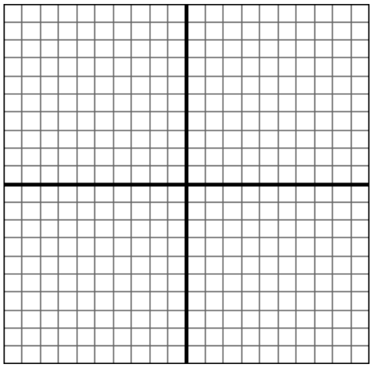


54. $2x-y<1$

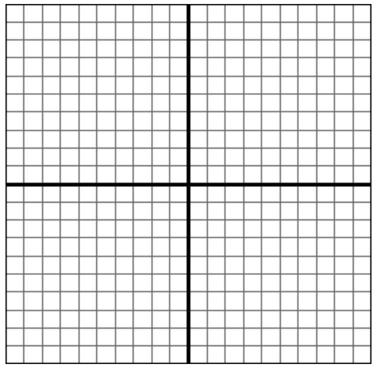


**Sketch the graph of each function.**

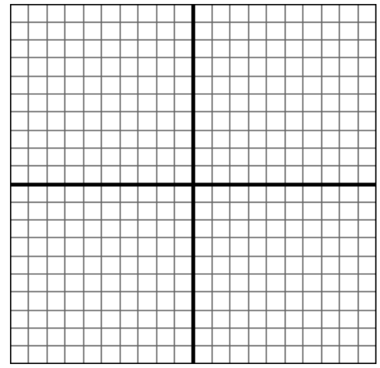
55. $y = -x^2 + 2x - 4$



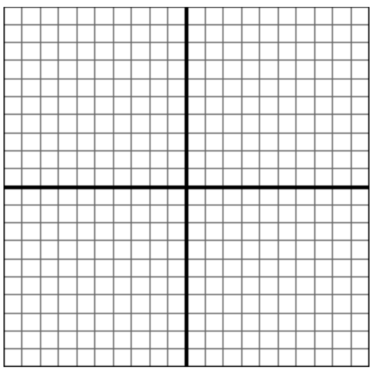
56. $y = 2x^2 + 16x + 12$



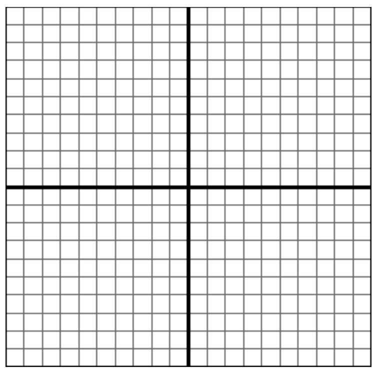
57. $y = -x^2 + 8x - 19$



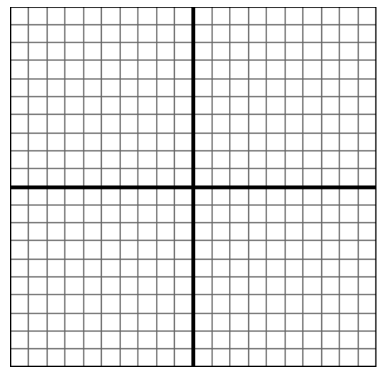
58. $y < -x^2 - 4x - 6$



59. $y > \frac{1}{2}x^2 - 2x + 4$

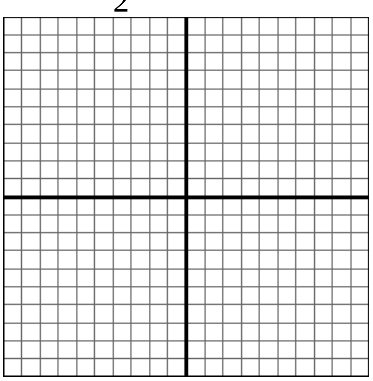


60. $y = x^2 - 4x + 5$

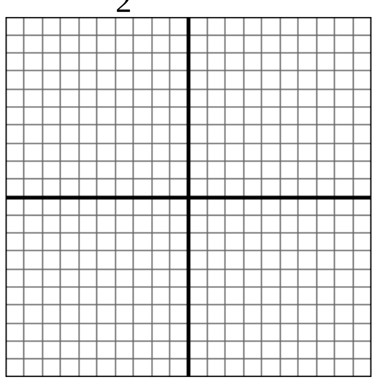


**Solve each system of equations or inequalities by graphing.**

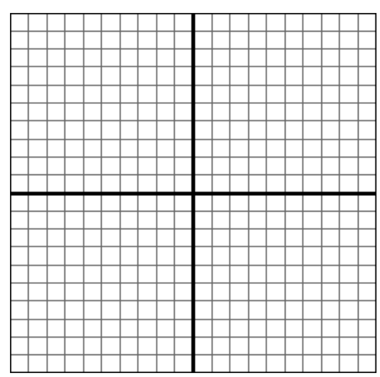
61. $y = -3x - 2$
$y = \frac{1}{2}x - 9$



62. $y = -x - 4$
$y = \frac{3}{2}x + 1$

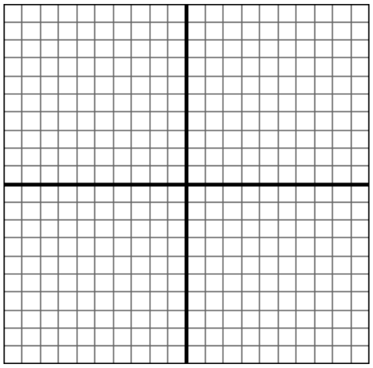


63. $y = -x - 9$
$y = x - 1$



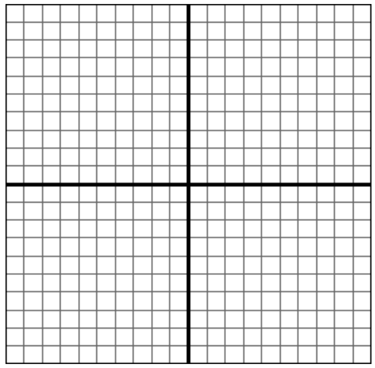
64. $y = -x - 7$

$y = \frac{5}{8}x + 6$



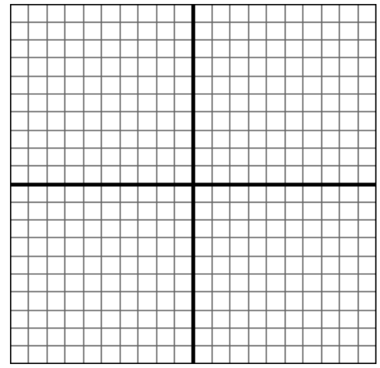
65. $y \ge 5x + 3$

$y < -2$



66. $y > 2x - 3$

$y \le \frac{1}{2}x + 2$



**Logarithms**

**Expanding and Condensing Logarithms**

Example 1: Expand the logarithm $\log_2 x^3 y^2 z$.

$$\log_2\left(\frac{\overbrace{x^3 \bullet y^2}^{\text{mult}\to\text{add in expansion}}}{\underbrace{\sqrt{z}}_{\text{div. by }\sqrt{z}\to\text{subt. in expansion}}}\right) = \log_2 \underbrace{x^3}_{} \underset{\text{from mult.}}{+} \log_2 y^2 \underset{\text{from div.}}{-} \log_2 \underset{\sqrt{z}=z^{\frac{1}{2}}}{z^{\frac{1}{2}}} = 3\log_2 x + 2\log_2 y - \frac{1}{2}\log_2 z$$

Example 2: Write as one logarithm (condense) $2\log_4 a + 3\log_4 b - 2\log_4 c$

$$2\log_4 a + 3\log_4 b - 2\log_4 c = \underbrace{\log_4 a^2 + \log_4 b^3 - \log_4 c^2}_{\text{move coefficients to exponents}} = \log_4 \underbrace{\left(a^2 \bullet b^3\right)}_{\text{add becomes mult}} - \log_4 c^2 = \log_4 \underbrace{\left(\frac{a^2 b^3}{c^2}\right)}_{\text{subt. becomes div.}}$$

**Expand or condense each logarithm.**

67. $\log_7\left(2 \bullet 3 \bullet 5^6\right)$

68. $\log_7\left(z^3\sqrt{x}\right)$

69. $\log_6\left(\frac{u^5}{v}\right)^4$

70. $5\log_3 a + 2\log_3 d$

71. $\frac{1}{2}\log x - 3\log y$

72. $2\log x + 5\log y - 3\log z$

**Solving Logarithmic Equations**

Example 1: Solve the equation $\log_2 x - \log_2(x+1) = \log_2 10$.

$$\log_2 x - \log_2(x+1) = \log_2 10 \quad\Rightarrow\quad \log_2\left(\frac{x}{x+1}\right) = \log_2 10 \quad\Rightarrow\quad \frac{x}{x+1} = 10 \;\Rightarrow$$

$$x = 10(x+1) \quad\Rightarrow\quad x = 10x + 10 \quad\Rightarrow\quad -9x = 10 \quad\Rightarrow\quad x = -\frac{10}{9}$$

**Solve each equation.**

73. $\log_8 6 - \log_8 (x-7) = \log_8 3$

74. $\log_6 x - \log_6 (x-2) = \log_6 45$

75. $\log_6 x - \log_6 (x-2) = 2$
(not solved the same as 73 & 74)

**Simplify each rational expression.**

76. $\dfrac{n-10}{n-9} \bullet \dfrac{n^2-10n+9}{n-1}$

77. $\dfrac{1}{m^2-49} \bullet \dfrac{m^2-m-42}{m+10}$

78. $\dfrac{x-4}{x^2-10x+24} \div \dfrac{x+1}{10x^3-60x^2}$

79. $\dfrac{n^2+11n+28}{n+7} \div \dfrac{9n+36}{4}$

**Solve each equation. Remember to check for extraneous solutions.**

80. $\dfrac{m-4}{5m} = \dfrac{1}{5m} - 1$

81. $\dfrac{1}{4r^2} = \dfrac{r+2}{2r^2} - \dfrac{1}{r^2}$

**Find the discriminant $b^2 - 4ac$ of each quadratic then state the number and type of solutions.**

82. $-r^2 - 6r - 2 = 7$

83. $9x^2 - 7x + 7 = -2$

**Radical Expressions**

**Simplify.**

84. $-3\sqrt{6}\left(5\sqrt{2}+5\right)$

85. $\sqrt{15}\left(-5\sqrt{5}-2\sqrt{3}\right)$

86. $-4\sqrt{15}\left(5+\sqrt{6}\right)$

87. $\sqrt{10}\left(2\sqrt{3}+\sqrt{2}\right)$

88. $\sqrt[7]{256x^3}$

89. $\sqrt[3]{-64m^3}$

90. $\sqrt[3]{512x^6}$

91. $\sqrt[4]{112x^5}$

**Write each expression in radical form.**

92. $x^{\frac{1}{2}}$

93. $(7n)^{\frac{3}{2}}$

94. $\left(x^2\right)^{\frac{2}{3}}$

**Simplify. Your answer should contain only positive exponents with no fractional exponents in the denominator.**

95. $4y^{\frac{1}{4}} \bullet 3x^{-1}$

96. $4u^{-2}v^{\frac{3}{4}} \bullet 3u^{\frac{5}{4}}v^{-2} \bullet u^{-2}v^{-2}$

**Find all rational zeros.**

97. $f(x) = x^3 + x^2 - 5x + 3$

98. $f(x) = x^3 + 9x^2 - 39x + 9$

**Approximate the real zeros of each function to the nearest tenth.**

99. $f(x) = -x^3 + 2x^2 - 2$

100. $f(x) = x^4 + x^3 - 4x^2 + 5$

**Evaluate each function for the given value.**

101. $f(n) = n^4 - 5n^3 + 11n^2 - 9n - 30$ find $f(3)$.

102. $f(x) = -6x^4 - 33x^3 - 10x^2 + 25x + 11$ find $f(-5)$.

103. $f(x) = x^2 + 5$ find $f(x-3)$.

104. $f(x) = 3x^2 - 5x + 1$ find $f(a+1)$.

**Solve the system of equations algebraically using elimination or substitution.**

105. $x + y = -4$
$-3x + 2y = 2$

106. $7x + 7y = -7$
$-2x - y = 8$

107. $-6x + 8y = 10$
$-10x + 4y = 26$

108. $3x + y = -9$
$-x + y = -1$

109. $y = 2x$
$y = x^2 + 3x - 2$

110. $x + y = 1$
$y = -x^2 - 2x + 3$